

INSTITUTE OF ACTUARIES OF INDIA
EXAMINATIONS

May 2026

CM1B- Actuarial Mathematics for Modelling

Time allowed: 1 Hours 45 Minutes

Total Marks: 100

Q.1 A life insurance company has issued a Regular Premium Endowment Assurance contract with a policy term of 15 years. The contract is issued to a policyholder aged 30 years exact at the time of entry. The level annual premium of Rs. 50000 is payable in advance, at the start of each policy year, until the earlier of death, surrender, or maturity.

Policy Benefits

1. Death Benefit: If the policyholder dies during the policy term, the benefit payable is:
 - 20 times the annual premium,
 - payable at the end of the policy year of death.
2. Maturity Benefit: If the policyholder survives to the end of the policy term, the maturity benefit payable is:
 - 20 times the annual premium,
 - payable at the end of the 15th policy year.
3. Surrender Benefit: If the policyholder surrenders the policy during the term, the surrender value payable at the end of the policy year of surrender is:
 - Surrender Value = Surrender value factor * Total premium paid to date
 - Surrender Value Factors are provided in the template.

Best Estimate Assumptions for Profit Testing

The company uses the following best estimate assumptions to carry out profit testing:

Demographic Assumptions

- Mortality Rates: As given in the template
- Surrender Rates: As given in the template

Both mortality and surrender rates are applied to policies in force at the start of the policy year.

Economic Assumptions

- Interest Rate: 6.5% per annum
- Risk Discount Rate: 12% per annum

Expense Assumptions

- Initial Expenses: INR 5,000 (incurred at policy inception)
- Renewal Expenses: INR 1,000 from the start of the second policy year onwards and inflates at 2.50% per annum from the start of the third policy year

Commission Assumptions

- Initial Commission: 25% of annual premium
- Renewal Commission: 5% of annual premium

Reserve Assumption (Formula-Based)

- Per policy reserves at the end of each policy year are calculated as:
- Reserve (Per Policy) = 45%*Total premium paid to date

Reserve at the end of policy term will be zero.

- i) Calculate the Profit Margin for this policy. [23]
- ii) In practice, the actuary uses a prudent assumption–based reserving approach rather than the above formula-based method. The actuary now intends to replace the formula-based reserves with reserves calculated using the following prudent assumptions:

Prudent Assumptions

- Mortality Rates: 120% of best estimate mortality rates
- Surrender Rates: Same as best estimate assumptions
- Interest Rate: 80% of best estimate interest rate
- Initial Expenses: Same as Best estimate expense
- Renewal Expenses: 120% of best estimate renewal expenses

Calculate the prospective per policy reserve at the end of each policy year using the prudent assumptions specified above. [10]

- iii) Recalculate the profit margin of the policy using the reserves calculated in part (ii), while keeping all other assumptions unchanged. [4]

- iv) Calculate the policyholder IRR, assuming that the policyholder:

- Pays all premiums as scheduled, and
- Survives until maturity and receives the maturity benefit.

[3]

- v) A company issues two types of temporary annuity policies to a couple consisting of a man aged 60 and a woman aged 57, Each policy provides annual payments of ₹50,000 in arrears for a maximum of 10 years, subject to survival conditions:

1. Joint-life temporary annuity - Payments continue until the earlier of:

- 10 years, or
- The first death of either the man or the woman.

2. Last-survivor temporary annuity - Payments continue until the earlier of:

- 10 years, or
- The second death (i.e., both the man and the woman must have died).

You are required to calculate the expected present value (EPV) of each annuity product, assuming:

- Mortality follows the given l_x tables for the man and woman respectively (as provided in the input sheet)

- Interest rate is 6% per annum effective throughout.

[7]

[47 marks]

Q.2) Mr. Daksh has taken an educational loan of INR 50 lakh for higher studies. The loan amount will be disbursed in four equal half-yearly instalments, payable in advance, over a period of 2 years. The first disbursement is made at the start of the loan (time 0). Interest is charged on each disbursed instalment from its respective date of disbursement.

The loan is repaid over a period of 10 years in equal monthly instalments. The first repayment instalment is paid at the end of the 25th month from the date of first loan disbursement. The rate of interest is 10% per annum effective, applicable throughout the loan term.

i) Calculate the amount of each monthly repayment instalment. [10]

ii) After 5 years of repayment, the student wishes to increase the monthly repayment instalment so that the outstanding loan amount is fully repaid over the next 2 years.

Calculate the new monthly instalment amount, assuming the same effective rate of interest as above and that repayments continue to be made monthly in arrears. [5]

iii) ABC Bank issues a bond with the following features:

- Coupon rate: 6% per annum, payable quarterly in arrears
- Term to maturity: 5 years
- Redemption: at par at the end of Year 5, together with the final coupon
- Face value: INR 1,00,000

An investor purchasing the bond is subject to:

- Income tax at 30% on each coupon payment, and
- Capital gains tax at 20% on the excess of the redemption value over the purchase price.

Calculate the maximum price the investor should be willing to pay for the bond if they require a net effective rate of return of 6.5% per annum, after allowing for taxation.

Assume:

- Taxes are payable at the time the corresponding cash flows are received, and
- The required rate of return is an annual effective rate.

[10]

[25 marks]

Q.3) An input sheet provides the value of a zero-coupon bond with face value 100 for maturities ranging from 1 year to 20 years. The bonds relate to Government of India securities. Assume all interest rates are annual effective, and all cash flows occur at the end of each year.

i) Using the data provided in the input sheet, calculate the spot rate (annual effective) for each maturity from Year 1 to Year 20. [5]

ii) Using the spot rates obtained in part (i), calculate the one-year forward rate for each period from Year 0–1 up to Year 19–20,

(Hint: The forward rate for Year 15–16 represents the one-year forward rate applicable at the end of Year 15.) **[5]**

iii) A bond has the following features:

- Coupon rate: 6% per annum, payable annually in arrears
- Term: 5 years
- Redemption: at par at the end of Year 5
- Face value: INR 1,00,000

Calculate the price of the bond:

- using the spot rates derived in part (i), and
- using the one-year forward rates derived in part (ii).

[10]

iv) Divesh plans to purchase a car at the end of five years and will require ₹15 lakh in real value terms at that time. To meet this objective, he intends to save a portion of his salary by investing in a recurring deposit scheme offered by the India Post Office, which provides an effective annual interest rate of 8.15%. The expected rate of inflation is 5% per annum. Divesh plans to increase his monthly deposit by 2% every month, starting from the second month. He will make his first deposit at the end of this month. Based on the above information, determine the amount of the first monthly deposit he should make.

[8]

[28 marks]
